section seqdefs

/Reference manual/Extended toolkit

1. Sequences

The 'definition' parts of this chapter are a formal specification of the sequence section of the tool-kit. The principle of definition before use has been observed in all cases. Furthermore, all definitions are explicit, so that the model conjecture for the whole of this section is trivially satisfied.

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$$\text{section } seqdefs \text{ parents } numdefs, setdefs$$

1.0.0.1. Name .. – Number range

1.0.0.2. Definition

$$\text{function } 20 \text{ leftassoc } (\_ \ldots \_)$$

$$\_ \ldots \_ == \lambda\, a, b : \mathbb{R} \bullet \{k : \mathbb{Z} \mid a \leq k \leq b\}$$

1.0.0.3. Name $\#$ – Number of members of a set

1.0.0.4. Definition

$$\#[X] == \{S : \mathbb{P}\, X;\ n : \mathbb{N} \mid 1 \mathinner{..} n \rightarrowtail\!\!\!\!\!\twoheadrightarrow S \neq \varnothing\}$$

1.0.0.5. Name $items$ – Bag of elements of an indexed set

1.0.0.6. Definition

$$items[I, X] == \lambda\, s : I \leftrightarrow X \mid \forall\, x : X \bullet finite(s \rhd \{x\}) \bullet \lambda\, x : ran\ s \bullet \#(s \rhd \{x\})$$

1.0.0.7. Name min, max – Minimum and maximum of a set of numbers

1.0.0.8. Definition

$$\begin{aligned} min == \{&S : \mathbb{P}_1\, \mathbb{R};\ m : \mathbb{R} \mid \\ &m \in S \land (\forall\, n : S \bullet m \leq n) \bullet S \mapsto m\} \end{aligned}$$

$$\begin{aligned} max == \{&S : \mathbb{P}_1\, \mathbb{R};\ m : \mathbb{R} \mid \\ &m \in S \land (\forall\, n : S \bullet m \geq n) \bullet S \mapsto m\} \end{aligned}$$

2. Sequences

2.0.0.9. Name

$sequence$	–	Sequences
seq	–	Finite sequences
$sequence_1$	–	Non-empty sequences
$isequence$	–	Injective sequences

2.0.0.10. Definition

$$\mathrm{generic}(sequence\ \_)$$

$$sequence X == \{f : \mathbb{N}_1 \pfun X \mid \forall n : \mathrm{dom}\, f;\ m : \mathbb{N}_1 \mid m < n \bullet m \in \mathrm{dom}\, f\}$$

$$sequence_1 X == \{f : sequence X \mid f \neq \varnothing\}$$

$$\mathrm{generic}(isequence\ \_)$$

$$isequence X == sequence X \cap (\mathbb{N}_1 \pinj X)$$

$$isequence_1 X == \{f : isequence X \mid f \neq \varnothing\}$$

$$\mathrm{generic}(seq\ \_)$$

$$seq\ X == \{f : sequence X \mid finite\ f\}$$

$$seq_1\ X == \{f : seq\ X \mid f \neq \varnothing\}$$

generic$(iseq\ _)$

$$iseq\ X == \{f : isequence X \mid finite\ f\}$$

$$iseq_1\ X == \{f : iseq\ X \mid f \neq \varnothing\}$$

2.0.0.11. Description $sequence X$ is the set of finite or infinite sequences over X. These are functions from $\mathbb{N}_1$ to X where for every element in the domain, all lower members of $\mathbb{N}_1$ are also in the domain. There are then three possible orthogonal restrictions on sequences, generating eight cases in all. The restrictions are: to finiteness, $seq\ X$ etc, to non-emptiness, $sequence_1 X$ etc, and to injectivity, $isequence X$ etc.

2.0.0.12. Name Finite Sequence Displays.

2.0.0.13. Definition

function $(\langle , , \rangle)$

$$\langle , , \rangle[X] == \lambda\, s : seq\ X \bullet s$$

2.0.0.14. Name $\frown$ – Concatenation

2.0.0.15. Definition

function 30 leftassoc $(\_ \frown \_)$

$$\_ \frown \_[X] == \lambda\, s : seq\ X;\ t : sequenceX \bullet s \cup \{n : dom\, t \bullet n + \#s \mapsto t\ \ n\}$$

2.0.0.16. Name rev – Reverse

2.0.0.17. Definition

$$rev[X] == \lambda\, s : seq\ X \bullet \{n : dom\, s \bullet \#\{i : dom\, s \mid n \leq i\} \mapsto s\ \ n\}$$

2.0.0.18. Name $head, last, tail, front$ – Sequence decomposition

2.0.0.19. Definition

$$head[X] == \lambda\, s : sequence_1 X \bullet s\ \ 1$$

$$last[X] == \lambda\, s : seq_1\ X \bullet s(\#s)$$

$$tail[X] == \lambda\, s : sequenceX \bullet \{m, n : dom\, s \mid n = m + 1 \bullet m \mapsto s\ \ n\}$$

$$front[X] == \lambda\, s : sequenceX \bullet \{m, n : dom\, s \mid n = m + 1 \bullet m \mapsto s\ \ m\}$$

2.0.0.20. Name

$squash$	–	Compaction
$\upharpoonleft$	–	Extraction
$\upharpoonright$	–	Filtering

2.0.0.21. Definition

$$squash[X] == \lambda f : \mathbb{N} \pfun X \bullet \{p : dom\, f \bullet \#\{i : dom\, f \mid i \leq p\} \mapsto f\ p\}$$

function 40 leftassoc $(\_ \upharpoonright \_)$

$$\_ \upharpoonright \_[X] == \lambda s : sequenceX;\ V : \mathbb{P}\, X \bullet squash(s \rhd V)$$

function 41 rightassoc $(\_ \upharpoonleft \_)$

$$\_ \upharpoonleft \_[X] == \lambda U : \mathbb{P}\, \mathbb{N};\ s : sequenceX \bullet squash(U \lhd s)$$

2.0.0.22. Name

$prefix$	–	Prefix relation
$suffix$	–	Suffix relation
infix	–	Segment relation

2.0.0.23. Definition

relation$(_\ prefix\ _)$

$$\_\,prefix\,\_[X] == \{s, t : sequence X \mid s \subseteq t\}$$

$$\text{relation}(\_\,suffix\,\_)$$

$$\_\,suffix\,\_[X] == \{s, t : sequence X \mid (\exists\, u : seq\ X \bullet u \frown s = t)\}$$

$$\text{relation}(\_\,\text{infix}\,\_)$$

$$\_\,\text{infix}\,\_[X] == \{s, t : sequence X \mid (\exists\, u : seq\ X \bullet u \frown s \subseteq t)\}$$

2.0.0.24. Name $+/$ – Distributed sum

2.0.0.25. Definition

$$\begin{array}{l} +/[I] == \bigcap\{f : (I \leftrightarrow \mathbb{R}) \leftrightarrow \mathbb{R} \mid \\ \quad (\varnothing\ , 0) \in f \land \\ \quad (\forall\, S : I \leftrightarrow \mathbb{R};\ p : I \times \mathbb{R};\ x : \mathbb{R} \mid \\ \quad\quad p \in S \land (S \setminus \{p\}, x) \in f \bullet \\ \quad\quad (S, x + p.2) \in f)\} \end{array}$$

2.0.0.26. Name $\frown/$ – Distributed concatenation

2.0.0.27. Definition

$$
\begin{array}{l}
{\frown}/[X] == \lambda\, s : sequencesequenceX \mid front\ \ s \in sequence\ seq\ X \bullet \\
\quad \{m : dom\ s;\ \ n : \mathbb{N}_1 \mid n \in dom(s\ \ m) \bullet \\
\qquad (n + +/(\lambda\, k : 1 \mathinner{\ldotp\ldotp} m - 1 \bullet \#(s\ \ k)), s\ \ m\ \ n)\}
\end{array}
$$

2.0.0.28. Name R^k – Iteration

2.0.0.29. Definition

$$
\begin{array}{l}
iter[X] == \bigcap\{f : \mathbb{Z} \leftrightarrow (X \leftrightarrow X) \leftrightarrow (X \leftrightarrow X) \mid \\
\quad (0, (\lambda\, R : X \leftrightarrow X \bullet id\ X)) \in f \land \\
\quad (\forall\, k : \mathbb{N};\ \ R, S : X \leftrightarrow X;\ \ g : (X \leftrightarrow X) \leftrightarrow (X \leftrightarrow X) \mid \\
\qquad (R, S) \in g \land (k, g) \in f \bullet \\
\qquad\quad \exists\, h : (X \leftrightarrow X) \leftrightarrow (X \leftrightarrow X) \bullet (R, R \mathbin{;} S) \in h \land (k + 1, h) \in f) \land \\
\quad (\forall\, k : \mathbb{N};\ \ R, S : X \leftrightarrow X;\ \ g : (X \leftrightarrow X) \leftrightarrow (X \leftrightarrow X) \mid \\
\qquad (R^{\sim}\ , S) \in g \land (k, g) \in f \bullet \\
\qquad\quad \exists\, h : (X \leftrightarrow X) \leftrightarrow (X \leftrightarrow X) \bullet (R, S) \in h \land (\text{-}k, h) \in f)\}
\end{array}
$$

2.0.0.30. Notation $iter\ \ k\ \ R$ is sometimes written R^k.

function($\_\ ^{-}$)

$$\_^{-}[X] == \lambda\, r : X \leftrightarrow X;\ \ n : \mathbb{N} \bullet iter\ \ n\ \ r$$