distribution

/Reference manual/Z-related commands/In situ replacement commands

There are two categories of distributions: elementary ones that are built-in to cadiz, and ones that are coded explicitly as Z rewrite rules. An example of the latter is as follows.

union_distribution ==

$$[X] \vdash ?\ \forall S, T, U : \mathbb{P}\, X \bullet S \cup (T \cup U) = (S \cup T) \cup U$$

The form of a rewrite rule must be as specified in *rewrite by rule*, and moreover its name must have *distribution* as a sub-string. The effect of a rewrite rule is also explained in *rewrite by rule*.

The built-in elementary distributions spread out, or distribute, the selected formula into the immediate parent formula of which it is a part. These distributions behave as *in situ* replacements of the parents. In each of the distribution rules, the left-hand side depicts the parent formula, with the inspected formula shown parenthesized. Only one distribution can be performed at once. They take precedence over any matching explicit rewrite rule. In each of the following subsections, the earliest applicable distribution in the list is the one that is applied.

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2. Predicate

2.1. Conjunction

$$
\begin{array}{rcl}
p_1 \land (p_2 \land p_3) & \Longrightarrow & (p_1 \land p_2) \land p_3 \\
(p_1 \land p_2) \land p_3 & \Longrightarrow & p_1 \land (p_2 \land p_3) \\
p_1 \land (p_2 \lor p_3) & \Longrightarrow & (p_1 \land p_2) \lor (p_1 \land p_3) \\
(p_1 \lor p_2) \land p_3 & \Longrightarrow & (p_1 \land p_3) \lor (p_2 \land p_3) \\
p_1 \land (\neg\, p_2) & \Longrightarrow & \neg\, (p_1 \Rightarrow p_2) \\
(\neg\, p_1) \land p_2 & \Longrightarrow & \neg\, (p_2 \Rightarrow p_1) \\
p_1 \land (p_2 \Rightarrow p_3) & \Longrightarrow & (p_1 \Rightarrow p_2) \Rightarrow (p_1 \land p_3) \\
(p_1 \Rightarrow p_2) \land p_3 & \Longrightarrow & (p_3 \Rightarrow p_1) \Rightarrow (p_2 \land p_3) \\
p_1 \land (p_2 \Leftrightarrow p_3) & \Longrightarrow & p_1 \land p_2 \Leftrightarrow p_1 \Rightarrow p_3 \\
(p_1 \Leftrightarrow p_2) \land p_3 & \Longrightarrow & p_3 \Rightarrow p_1 \Leftrightarrow p_2 \land p_3 \\
p_1 \land (\exists\, s \bullet p_2) & \Longrightarrow & \exists\, s \bullet p_1 \land p_2 \\
(\exists\, s \bullet p_1) \land p_2 & \Longrightarrow & \exists\, s \bullet p_1 \land p_2 \\
p_1 \land (\exists_1\, s \bullet p_2) & \Longrightarrow & \exists_1\, s \bullet p_1 \land p_2 \\
(\exists_1\, s \bullet p_1) \land p_2 & \Longrightarrow & \exists_1\, s \bullet p_1 \land p_2
\end{array}
$$

2.2. Disjunction

$$
\begin{array}{rcl}
p_1 \lor (p_2 \land p_3) & \Longrightarrow & (p_1 \lor p_2) \land (p_1 \lor p_3) \\
(p_1 \land p_2) \lor p_3 & \Longrightarrow & (p_1 \lor p_3) \land (p_2 \lor p_3) \\
p_1 \lor (p_2 \lor p_3) & \Longrightarrow & (p_1 \lor p_2) \lor p_3 \\
(p_1 \lor p_2) \lor p_3 & \Longrightarrow & p_1 \lor (p_2 \lor p_3) \\
p_1 \lor (\lnot p_2) & \Longrightarrow & p_2 \Rightarrow p_1 \\
(\lnot p_1) \lor p_2 & \Longrightarrow & p_1 \Rightarrow p_2 \\
p_1 \lor (p_2 \Rightarrow p_3) & \Longrightarrow & p_2 \Rightarrow (p_1 \lor p_3) \\
(p_1 \Rightarrow p_2) \lor p_3 & \Longrightarrow & p_1 \Rightarrow (p_2 \lor p_3) \\
p_1 \lor (p_2 \Leftrightarrow p_3) & \Longrightarrow & p_2 \Rightarrow p_1 \Leftrightarrow p_3 \Rightarrow p_1 \\
(p_1 \Leftrightarrow p_2) \lor p_3 & \Longrightarrow & p_1 \Rightarrow p_3 \Leftrightarrow p_2 \Rightarrow p_3 \\
p_1 \lor (\forall s \bullet p_2) & \Longrightarrow & \forall s \bullet p_1 \lor p_2 \\
(\forall s \bullet p_1) \lor p_2 & \Longrightarrow & \forall s \bullet p_1 \lor p_2
\end{array}
$$

2.3. Negation

$$
\begin{array}{rcl}
\lnot (p_1 \land p_2) & \Longrightarrow & \lnot p_1 \lor \lnot p_2 \\
\lnot (p_1 \lor p_2) & \Longrightarrow & \lnot p_1 \land \lnot p_2 \\
\lnot (p_1 \Rightarrow p_2) & \Longrightarrow & p_1 \land \lnot p_2 \\
\lnot (p_1 \Leftrightarrow p_2) & \Longrightarrow & \lnot p_1 \Leftrightarrow p_2
\end{array}
$$

2.4. Implication

$$
\begin{array}{rcl}
p_1 \Rightarrow (p_2 \land p_3) & \Longrightarrow & (p_1 \Rightarrow p_2) \land (p_1 \Rightarrow p_3) \\
(p_1 \land p_2) \Rightarrow p_3 & \Longrightarrow & p_1 \Rightarrow (p_2 \Rightarrow p_3) \\
p_1 \Rightarrow (p_2 \lor p_3) & \Longrightarrow & (p_1 \Rightarrow p_2) \lor p_3 \\
(p_1 \lor p_2) \Rightarrow p_3 & \Longrightarrow & (p_1 \Rightarrow p_3) \land (p_2 \Rightarrow p_3) \\
p_1 \Rightarrow (\lnot\, p_2) & \Longrightarrow & \lnot\, (p_1 \land p_2) \\
(\lnot\, p_1) \Rightarrow p_2 & \Longrightarrow & p_1 \lor p_2 \\
p_1 \Rightarrow (p_2 \Rightarrow p_3) & \Longrightarrow & (p_1 \land p_2) \Rightarrow p_3 \\
(p_1 \Rightarrow p_2) \Rightarrow p_3 & \Longrightarrow & (p_1 \lor p_3) \land (p_2 \Rightarrow p_3) \\
p_1 \Rightarrow (p_2 \Leftrightarrow p_3) & \Longrightarrow & (p_1 \land p_2) \Leftrightarrow (p_1 \land p_3) \\
(p_1 \Leftrightarrow p_2) \Rightarrow p_3 & \Longrightarrow & (p_1 \lor p_3) \Leftrightarrow (p_2 \Rightarrow p_3) \\
p_1 \Rightarrow (\forall\, s \bullet p_2) & \Longrightarrow & \forall\, s \bullet p_1 \Rightarrow p_2 \\
(\exists\, s \bullet p_1) \Rightarrow p_2 & \Longrightarrow & \forall\, s \bullet p_1 \Rightarrow p_2
\end{array}
$$

2.5. Equivalence

$$
\begin{array}{rcl}
p_1 \Leftrightarrow (\lnot\, p_2) & \Longrightarrow & \lnot\, (p_1 \Leftrightarrow p_2) \\
(\lnot\, p_1) \Leftrightarrow p_2 & \Longrightarrow & \lnot\, (p_1 \Leftrightarrow p_2) \\
p_1 \Leftrightarrow (p_2 \Leftrightarrow p_3) & \Longrightarrow & (p_1 \Leftrightarrow p_2) \Leftrightarrow p_3 \\
(p_1 \Leftrightarrow p_2) \Leftrightarrow p_3 & \Longrightarrow & p_1 \Leftrightarrow (p_2 \Leftrightarrow p_3)
\end{array}
$$

2.6. Universal quantification

$$
\begin{array}{rcl}
\forall\, ds \mid (p_1 \lor p_2) \bullet p_3 & \Longrightarrow & (\forall\, ds \mid p_1 \bullet p_3) \land (\forall\, ds \mid p_2 \bullet p_3) \\
\forall\, s \bullet (p_1 \land p_2) & \Longrightarrow & (\forall\, s \bullet p_1) \land (\forall\, s \bullet p_2) \\
\forall\, s \bullet (p_1 \lor p_2) & \Longrightarrow & (\forall\, s \bullet p_1) \lor (\forall\, s \bullet p_2) \text{ where } s \text{ has only} == \text{decs} \\
\forall\, s \bullet (p_1 \Rightarrow p_2) & \Longrightarrow & (\exists\, s \bullet p_1) \Rightarrow (\forall\, s \bullet p_2) \text{ where } s \text{ has only} == \text{decs}
\end{array}
$$

See also *separation*.

2.7. Existential quantification

$$
\begin{array}{rcl}
\exists\, ds \mid (p_1 \lor p_2) \bullet p_3 & \Longrightarrow & (\exists\, ds \mid p_1 \bullet p_3) \lor (\exists\, ds \mid p_2 \bullet p_3) \\
\exists\, s \bullet (p_1 \land p_2) & \Longrightarrow & (\exists\, s \bullet p_1) \land (\exists\, s \bullet p_2) \text{ where } s \text{ has only} == \text{decs} \\
\exists\, s \bullet (p_1 \lor p_2) & \Longrightarrow & (\exists\, s \bullet p_1) \lor (\exists\, s \bullet p_2) \\
\exists\, s \bullet (p_1 \Rightarrow p_2) & \Longrightarrow & (\forall\, s \bullet p_1) \Rightarrow (\exists\, s \bullet p_2)
\end{array}
$$

See also *separation*.

2.8. Unique existential quantification

$$
\begin{array}{rcl}
\exists_1\, s \bullet (p_1 \land p_2) & \Longrightarrow & (\exists_1\, s \bullet p_1) \land (\exists_1\, s \bullet p_2) \text{ where } s \text{ has only} == \text{decs} \\
\exists_1\, s \bullet (p_1 \lor p_2) & \Longrightarrow & (\exists_1\, s \bullet p_1) \lor (\exists_1\, s \bullet p_2) \text{ where } s \text{ has only} == \text{decs} \\
\exists_1\, s \bullet (p_1 \Rightarrow P_2) & \Longrightarrow & (\exists_1\, s \bullet p_1) \Rightarrow (\exists_1\, s \bullet p_2) \text{ where } s \text{ has only} == \text{decs}
\end{array}
$$

See also *separation*.

2.9. Relation

$$
\begin{aligned}
relop\ (\ldots,\ (if\ p\ then\ e_1\ else\ e_2),\ \ldots\ ) &\quad\Longrightarrow\quad (p \Rightarrow relop\ (\ldots, e_1, \ldots)) \wedge (\neg\ p \Rightarrow relop\ (\\
relop\ (\ldots, (\mu\ s \bullet e), \ldots) &\quad\Longrightarrow\quad \exists\ s \bullet relop\ (\ldots, e, \ldots) \text{ where } s \text{ has only } = \\
e_1 \in ds \mid (p_1 \vee p_2) \bullet e_2 &\quad\Longrightarrow\quad e_1 \in ds \mid p_1 \bullet e_2 \vee e_1 \in ds \mid p_2 \bullet e_2 \\
e_1 \in \lambda\ ds \mid (p_1 \vee p_2) \bullet e_2 &\quad\Longrightarrow\quad e_1 \in \lambda\ ds \mid p_1 \bullet e_2 \vee e_1 \in \lambda\ ds \mid p_2 \bullet e_2 \\
e \in (e_1 \wedge e_2) &\quad\Longrightarrow\quad e \in e_1 \wedge e \in e_2 \text{ where } e_1 \text{ and } e_2 \text{ are of t} \\
e \in (e_1 \vee e_2) &\quad\Longrightarrow\quad e \in e_1 \vee e \in e_2 \text{ where } e_1 \text{ and } e_2 \text{ are of t} \\
e \in (e_1 \Rightarrow e_2) &\quad\Longrightarrow\quad e \in e_1 \Rightarrow e \in e_2 \text{ where } e_1 \text{ and } e_2 \text{ are of} \\
e \in (e_1 \Leftrightarrow e_2) &\quad\Longrightarrow\quad e \in e_1 \Leftrightarrow e \in e_2 \text{ where } e_1 \text{ and } e_2 \text{ are of} \\
e \in (\neg\ e_2) &\quad\Longrightarrow\quad \neg\ e \in e_2
\end{aligned}
$$

3. Expression

3.1. Schema construction

$$
\begin{aligned}
[ds \mid (p_1 \wedge p_2)] &\quad\Longrightarrow\quad [ds \mid p_1] \wedge [ds \mid p_2] \\
[ds \mid (p_1 \vee p_2)] &\quad\Longrightarrow\quad [ds \mid p_1] \vee [ds \mid p_2] \\
[ds \mid (p_1 \Rightarrow p_2)] &\quad\Longrightarrow\quad [ds \mid p_1] \Rightarrow [ds \mid p_2] \\
[ds \mid (p_1 \Leftrightarrow p_2)] &\quad\Longrightarrow\quad [ds \mid p_1] \Leftrightarrow [ds \mid p_2] \\
[ds \mid (\neg\ p)] &\quad\Longrightarrow\quad \neg\ [ds \mid p]
\end{aligned}
$$

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3.2. Schema decoration

$$
\begin{aligned}
(\langle\!|\ i_1 == e_1, \ldots, i_n == e_n\ |\!\rangle)+ &\quad\Longrightarrow\quad \langle\!|\ i_1+ == e_1, \ldots, i_n+ == e_n\ |\!\rangle \\
([d_1;\ \ldots;\ d_n \mid p])+ &\quad\Longrightarrow\quad [d_1+;\ \ldots;\ d_n+ \mid p+] \\
(\neg\ e)+ &\quad\Longrightarrow\quad \neg\ (e+) \\
(e_1 \wedge e_2)+ &\quad\Longrightarrow\quad (e_1+) \wedge (e_2+) \\
(e_1 \vee e_2)+ &\quad\Longrightarrow\quad (e_1+) \vee (e_2+) \\
(e_1 \Rightarrow e_2)+ &\quad\Longrightarrow\quad (e_1+) \Rightarrow (e_2+) \\
(e_1 \Leftrightarrow e_2)+ &\quad\Longrightarrow\quad (e_1+) \Leftrightarrow (e_2+) \\
(e_1 \veebar e_2)+ &\quad\Longrightarrow\quad (e_1+) \veebar (e_2+)
\end{aligned}
$$

3.3. Schema renaming

$$
\begin{aligned}
([d_1;\ \ldots;\ d_n \mid p])[\ldots, j_k/i_k, \ldots] &\quad\Longrightarrow\quad [d_1[\ldots, j_k/i_k, \ldots];\ \ldots;\ d_n[\ldots, j_k/i_k, \ldots] \mid p[\ldots, j_k/i_k, \ldots \\
(\neg\ e)[\ldots, j_k/i_k, \ldots] &\quad\Longrightarrow\quad \neg\ (e[\ldots, j_k/i_k, \ldots]) \\
(e_1 \wedge e_2)[\ldots, j_k/i_k, \ldots] &\quad\Longrightarrow\quad (e_1[\ldots, j_k/i_k, \ldots]) \wedge (e_2[\ldots, j_k/i_k, \ldots]) \\
(e_1 \vee e_2)[\ldots, j_k/i_k, \ldots] &\quad\Longrightarrow\quad (e_1[\ldots, j_k/i_k, \ldots]) \vee (e_2[\ldots, j_k/i_k, \ldots]) \\
(e_1 \Rightarrow e_2)[\ldots, j_k/i_k, \ldots] &\quad\Longrightarrow\quad (e_1[\ldots, j_k/i_k, \ldots]) \Rightarrow (e_2[\ldots, j_k/i_k, \ldots]) \\
(e_1 \Leftrightarrow e_2)[\ldots, j_k/i_k, \ldots] &\quad\Longrightarrow\quad (e_1[\ldots, j_k/i_k, \ldots]) \Leftrightarrow (e_2[\ldots, j_k/i_k, \ldots]) \\
(e_1 \veebar e_2)[\ldots, j_k/i_k, \ldots] &\quad\Longrightarrow\quad (e_1[\ldots, j_k/i_k, \ldots]) \veebar (e_2[\ldots, j_k/i_k, \ldots])
\end{aligned}
$$

3.4. Schema precondition

$$
\mathrm{pre}(e_1 \vee e_2) \quad\Longrightarrow\quad \mathrm{pre}\ e_1 \vee \mathrm{pre}\ e_2
$$

3.5. Schema quantification

$$
\begin{array}{rcl}
\forall\ ds_1 \mid p_1 \bullet ([ds_2 \mid p_2]) & \implies & [ds_2 \setminus ds_1 \mid \forall\ ds_1 \mid p_1 \bullet p_2] \\
\exists\ ds_1 \mid p_1 \bullet ([ds_2 \mid p_2]) & \implies & [ds_2 \setminus ds_1 \mid \exists\ ds_1 \mid p_1 \bullet p_2] \\
\exists_1\ ds_1 \mid p_1 \bullet ([ds_2 \mid p_2]) & \implies & [ds_2 \setminus ds_1 \mid \exists_1\ ds_1 \mid p_1 \bullet p_2]
\end{array}
$$

where there are no schema inclusions in ds_1 or ds_2.

4. Schema text

$$...;\ ([ds \mid p_1]);\ ... \mid p_2 \quad \implies \quad ...;\ ds;\ ... \mid p_1 \wedge p_2$$

where the selection is a schema inclusion declaration, and the schema text is not immediately within a lambda expression.

5. Sequent

$$...;\ ([ds \mid p]);\ ... \mid ps \vdash ?\ qs \quad \implies \quad ...;\ ds;\ ... \mid p, ps \vdash ?\ qs$$

where the selection is a schema inclusion declaration.

6. Tactic example

"distribution" p

This example applies the *distribution* command to predicate p.

Any jokers bound to the selected formula are rebound to the whole result, where those are both of the same syntactic category.

IT 6-Jul-2000