section seqlaws

/Reference manual/Extended toolkit

1. Laws about Sequences in The Mathematical Tool-kit

section *seqlaws* parents *seqdefs*, *numlaws*, *setlaws*

This section may eventually contain general-purpose tactics, and does contain laws, with (some of) their proofs, on the assumption that the definitions of seqdefs.z are present.

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1.1. Laws

L134 ==

$$\vdash?\ \forall a, b : \mathbb{R} \mid a > b \bullet a \mathinner{..} b = \varnothing$$

L135 ==

$$\vdash?\ \forall a : \mathbb{Z} \bullet a \mathinner{..} a = \{a\}$$

L136 ==

$$\vdash ? \ \forall a, b, c, d : \mathbb{R} \mid a \leq b \land c \leq d \bullet b \mathinner{\ldotp\ldotp} c \subseteq a \mathinner{\ldotp\ldotp} d$$

domainOfHash ==

$$[X] \vdash ? \ \mathit{dom}\, \# = \mathbb{F}\, X$$

L151 ==

$$[X] \vdash ? \ \forall S, T : \mathbb{F}\, X \bullet \#(S \cup T) + \#(S \cap T) = \#S + \#T$$

L152 ==

$$[X] \vdash ? \ \mathbb{F}_1\, X = \{S : \mathbb{F}\, X \mid 0 < \#S\}$$

L205 ==

$$[I, X] \vdash ? \ \forall s : I \leftrightarrow X \bullet \mathit{dom}(\mathit{items}\ s) = \mathit{ran}\, s$$

L206 ==

$$[I, X] \vdash ? \ \forall s, t : I \mathrel{+\!\!\!\!\leftrightarrow} X \bullet$$
$$\mathit{items}\ s = \mathit{items}\ t \Leftrightarrow (\exists f : \mathit{dom}\, s \rightarrowtail\!\!\!\!\twoheadrightarrow \mathit{dom}\, t \bullet s = t \circ f)$$

L154 ==

$\vdash ?\ \mathbb{F}_1\,\mathbb{Z} \subseteq dom\ min$

L155 ==

$\vdash ?\ \mathbb{F}_1\,\mathbb{Z} \subseteq dom\ max$

L156 ==

$\vdash ?\ (\mathbb{P}\,\mathbb{N}) \cap (dom\ min) = \mathbb{P}_1\,\mathbb{N}$

L157 ==

$\vdash ?\ (\mathbb{P}\,\mathbb{N}) \cap (dom\ max) = \mathbb{F}_1\,\mathbb{N}$

L158 ==

$\vdash ?\ \forall S, T : dom\ min \bullet min(S \cup T) = min\{min\ \ S, min\ \ T\}$

L159 ==

$\vdash ?\ \forall S, T : dom\ max \bullet max(S \cup T) = max\{max\ \ S, max\ \ T\}$

L160 ==

$\vdash ?\ \forall S, T : dom\ min \mid S \subseteq T \bullet min\ \ S \geq min\ \ T$

L161 ==

$$\vdash ?\ \forall S, T : dom\ max \mid S \subseteq T \bullet max\ S \leq max\ T$$

L162a ==

$$\vdash ?\ \forall a, b : \mathbb{Z} \mid a \leq b \bullet min(a \mathrel{..} b) = a$$

L162b ==

$$\vdash ?\ \forall a, b : \mathbb{Z} \mid a \leq b \bullet max(a \mathrel{..} b) = b$$

L163 ==

$$\vdash ?\ \forall a, b, c, d : \mathbb{Z} \bullet (a \mathrel{..} b) \cap (c \mathrel{..} d) = max\{a, c\} \mathrel{..} min\{b, d\}$$

L164 ==

$$[X] \vdash ?\ seq_1\ X = seq\ X \setminus \{\langle\rangle\}$$

L199 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}\ X \bullet disjoint \langle A \rangle$$

L200 ==

$$[X] \vdash ?\ \forall A, B : \mathbb{P}\ X \bullet disjoint \langle A, B \rangle \Leftrightarrow A \cap B = \varnothing$$

L201 ==

$$[X] \vdash ?\ \forall A, B, C : \mathbb{P}\, X \bullet$$
$$\langle A, B \rangle\ \mathit{partition}\ C \Leftrightarrow A \cap B = \varnothing \land A \cup B = C$$

L165 ==

$$[X] \vdash ?\ \forall s, t : \mathit{seq}\ X;\ u : \mathit{sequenceX} \bullet (s \frown t) \frown u = s \frown (t \frown u)$$

L166 ==

$$[X] \vdash ?\ \forall s : \mathit{sequenceX} \bullet \langle\rangle \frown s = s$$

L167 ==

$$[X] \vdash ?\ \forall s : \mathit{seq}\ X \bullet s \frown \langle\rangle = s$$

L168 ==

$$[X] \vdash ?\ \forall s, t : \mathit{seq}\ X \bullet \#(s \frown t) = \#s + \#t$$

S0 ==

$$[X, Y] \vdash ?\ \forall s : \mathit{sequenceX};\ f : X \rightarrow Y \bullet f \circ s \in \mathit{sequenceY}$$

S1 ==

$$[X, Y] \vdash ?\ \forall s : \mathit{seq}\ X;\ f : X \rightarrow Y \bullet \#(f \circ s) = \#s$$

S2 ==

$$[X, Y] \vdash ?\ \forall s : sequence X;\ f : X \rightarrow Y \bullet \forall i : dom\ s \bullet (f \circ s) i = f (s\ \ i)$$

S3 ==

$$[X, Y] \vdash ?\ \forall f : X \rightarrow Y \bullet f \circ \langle\rangle = \langle\rangle$$

S4 ==

$$[X, Y] \vdash ?\ \forall x : X;\ f : X \rightarrow Y \bullet f \circ \langle x \rangle = \langle f (x) \rangle$$

S5 ==

$$[X, Y] \vdash ?\ \forall s : seq\ X;\ t : sequence X;\ f : X \rightarrow Y \bullet f \circ (s \frown t) = (f \circ s) \frown (f \circ t)$$

S6 ==

$$[X] \vdash ?\ \forall s : sequence X \bullet ran\ s = \{ i : dom\ s \bullet s\ \ i \}$$

S7 ==

$$[X] \vdash ?\ \ ran \langle\rangle = \varnothing[X]$$

S8 ==

$$[X] \vdash ?\ \forall x : X \bullet ran \langle x \rangle = \{ x \}$$

S9 ==

$$[X] \vdash ? \ \forall s : seq\ X;\ t : sequenceX \bullet ran(s \frown t) = (ran\ s) \cup (ran\ t)$$

L169 ==

$$[X] \vdash ? \ rev[X]\langle\rangle = \langle\rangle$$

L170 ==

$$[X] \vdash ? \ \forall x : X \bullet rev\langle x\rangle = \langle x\rangle$$

L171 ==

$$[X] \vdash ? \ \forall s, t : seq\ X \bullet rev(s \frown t) = (rev\ \ t) \frown (rev\ \ s)$$

L172 ==

$$[X] \vdash ? \ \forall s : seq\ X \bullet rev(rev\ \ s) = s$$

S10 ==

$$[X, Y] \vdash ? \ \forall s : seq\ X;\ f : X \rightarrow Y \bullet rev(f \circ s) = f \circ (rev\ \ s)$$

S11 ==

$$[X] \vdash ? \ \forall s : seq\ X \bullet ran(rev\ \ s) = ran\ s$$

L173 ==

$$[X] \vdash ? \ \forall x : X \bullet head \langle x \rangle = last \langle x \rangle = x$$

L174 ==

$$[X] \vdash ? \ \forall x : X \bullet tail \langle x \rangle = front \langle x \rangle = \langle \rangle$$

L175a ==

$$[X] \vdash ? \ \forall s : seq_1 X; \ t : sequence X \bullet head(s \frown t) = head \ s$$

L175b ==

$$[X] \vdash ? \ \forall s : seq_1 X; \ t : sequence X \bullet tail(s \frown t) = (tail \ s) \frown t$$

L176a ==

$$[X] \vdash ? \ \forall s : seq X; \ t : seq_1 X \bullet last(s \frown t) = last \ t$$

L176b ==

$$[X] \vdash ? \ \forall s : seq X; \ t : seq_1 X \bullet front(s \frown t) = s \frown (front \ t)$$

L177 ==

$$[X] \vdash ? \ \forall s : sequence_1 X \bullet \langle head \ s \rangle \frown (tail \ s) = s$$

L178 ==

$$[X] \vdash ?\ \forall s : seq_1\, X \bullet (front\ \ s) \frown \langle last\ \ s \rangle = s$$

L179a ==

$$[X] \vdash ?\ \forall s : seq_1\, X \bullet head(rev\ \ s) = last\ \ s$$

L179b ==

$$[X] \vdash ?\ \forall s : seq_1\, X \bullet tail(rev\ \ s) = rev(front\ \ s)$$

L180a ==

$$[X] \vdash ?\ \forall s : seq_1\, X \bullet last(rev\ \ s) = head\ \ s$$

L180b ==

$$[X] \vdash ?\ \forall s : seq_1\, X \bullet front(rev\ \ s) = rev(tail\ \ s)$$

L181a ==

$$[X] \vdash ?\ \forall V : \mathbb{P}\, X \bullet \langle\rangle \upharpoonright V = \langle\rangle$$

L181b ==

$$[X] \vdash ?\ \forall U : \mathbb{P}\,\mathbb{N};\ s : seq\, X \mid s = \langle\rangle \bullet U \upharpoonleft s = \langle\rangle$$

L182 ==

$$[X] \vdash ?\ \forall s : seq\ X;\ t : sequenceX;\ V : \mathbb{P}\ X \bullet (s \frown t) \upharpoonright V = (s \upharpoonright V) \frown (t \upharpoonright V)$$

L183 ==

$$[X] \vdash ?\ \forall s : sequenceX;\ V : \mathbb{P}\ X \bullet ran\ s \subseteq V \Leftrightarrow s \upharpoonright V = s$$

L184a ==

$$[X] \vdash ?\ \forall s : sequenceX \bullet s \upharpoonright \varnothing = \langle\rangle$$

L184b ==

$$[X] \vdash ?\ \forall s : sequenceX \bullet \varnothing \upharpoonleft s = \langle\rangle$$

L185 ==

$$[X] \vdash ?\ \forall s : seq\ X;\ V : \mathbb{P}\ X \bullet \#(s \upharpoonright V) \leq \#s$$

L186 ==

$$[X] \vdash ?\ \forall s : seq\ X;\ V : \mathbb{P}\ X \bullet (rev\ \ s) \upharpoonright V = rev(s \upharpoonright V)$$

L187 ==

$$[X] \vdash ?\ \forall s : sequenceX;\ V, W : \mathbb{P}\ X \bullet (s \upharpoonright V) \upharpoonright W = s \upharpoonright (V \cap W)$$

S12 ==

$$[X, Y] \vdash ?\ \forall s : sequenceX;\ f : X \rightarrow Y;\ V : \mathbb{P}\ Y \bullet (f \circ s) \upharpoonright V = f \circ (s \upharpoonright f^{\sim}(\!|\ V\ |\!))$$

S13 ==

$$[X] \vdash ?\ \forall s : sequenceX;\ V : \mathbb{P}\,X \bullet ran(s \upharpoonright V) = (ran\ s) \cap V$$

L188 ==

$$[X] \vdash ?\ \forall s, t : sequenceX \bullet s\ prefix\ t \Leftrightarrow s = dom\ s \upharpoonleft t$$

L189 ==

$$[X] \vdash ?\ \forall s, t : seq\ X \bullet s\ suffix\ t \Leftrightarrow s = (\#t - \#s + 1 \,..\, \#t) \upharpoonleft t$$

L190 ==

$$[X] \vdash ?\ \forall s, t : seq\ X \bullet s\,\mathrm{infix}\,t \Leftrightarrow (\exists\, n : 1 \,..\, \#t \bullet s = (n \,..\, n + \#s) \upharpoonleft t)$$

L191 ==

$$[X] \vdash ?\ \forall s, t : sequenceX \bullet s\,\mathrm{infix}\,t \Leftrightarrow (\exists\, u : seq\ X \bullet s\ suffix\ u \wedge u\ prefix\ t)$$

L192 ==

$$[X] \vdash ?\ \forall s, t : sequenceX \bullet s\,\mathrm{infix}\,t \Leftrightarrow (\exists\, v : seq\ X \bullet s\ prefix\ v \wedge v\ suffix\ t)$$

dsumIsFunction ==

$$[I] \vdash ?\ +/ \in \mathbb{F}(I \times \mathbb{R}) \rightarrow \mathbb{R}$$

dcatOldDef1 ==

$$[X] \vdash ?\ \forall s : \mathit{seq}\ \mathit{seq}\ X \mid s = \langle\rangle \bullet {\frown}/\ s = \langle\rangle$$

dcatOldDef2 ==

$$[X] \vdash ?\ \forall s : \mathit{sequenceX} \bullet {\frown}/\langle s\rangle = s$$

dcatOldDef3 ==

$$[X] \vdash ?\ \forall q, r : \mathit{seq}\ \mathit{seq}\ X \bullet {\frown}/(q \frown r) = ({\frown}/\ q) \frown ({\frown}/\ r)$$

L193 ==

$$[X] \vdash ?\ \forall s, t : \mathit{seq}\ X \bullet {\frown}/\langle s, t\rangle = s \frown t$$

L194 ==

$$[X] \vdash ?\ \forall q : \mathit{seq}(\mathit{seq}\ X) \bullet \mathit{rev}({\frown}/\ q) = {\frown}/(\mathit{rev}(\mathit{rev} \circ q))$$

L195 ==

$$[X] \vdash ?\ \forall q : \mathit{seq}(\mathit{seq}\ X);\ V : \mathbb{P}\, X \bullet$$
$$({\frown}/\ q) \upharpoonright V = {\frown}/((\lambda\, s : \mathit{seq}\ X \bullet s \upharpoonright V) \circ q)$$

L196 ==

$$[X, Y] \vdash ?\ \forall f : X \leftrightarrow Y;\ q : \mathit{seq}(\mathit{seq}\ X) \bullet$$
$$f \circ ({\frown}/\ q) = {\frown}/((\lambda\, s : \mathit{seq}\ X \bullet f \circ s) \circ q)$$

L197 ==

$$[X] \vdash ?\ \forall q : seq(seq\ X) \bullet$$
$$ran(\frown / q) = \bigcup\{i : dom\ q \bullet ran(q\ i)\} = \bigcup(ran(ran \circ q))$$

L137 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet iter\ 0\ R = id\ X$$

L138 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet iter\ 1\ R = R$$

L139 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet iter\ 2\ R = R \mathbin{;} R$$

L140 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet iter(\text{-}1)R = R^{\sim}$$

L141 ==

$$[X] \vdash ?\ \forall k : \mathbb{N};\ R : X \leftrightarrow X \bullet iter(k+1)R = R \mathbin{;} iter\ k\ R = iter\ k\ R \mathbin{;} R$$

L142 ==

$$[X] \vdash ?\ \forall a : \mathbb{Z};\ R : X \leftrightarrow X \bullet iter\ a(R^{\sim}) = (iter\ a\ R)^{\sim}$$

L143 ==

$$[X] \vdash ?\ \forall a, b : \mathbb{N};\ R : X \leftrightarrow X \bullet iter(a + b)R = iter\ a\ R \mathbin{;} iter\ b\ R$$

L144 ==

$$[X] \vdash ?\ \forall a, b : \mathbb{Z};\ R : X \leftrightarrow X \bullet iter(a * b)R = iter\ b(iter\ a\ R)$$

L145 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet R^{+} = \bigcup\{k : \mathbb{N}_1 \bullet iter\ k\ R\}$$

L146 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet R^{*} = \bigcup\{k : \mathbb{N} \bullet iter\ k\ R\}$$

L147 ==

$$[X] \vdash ?\ \forall a : \mathbb{Z};\ R, S : X \leftrightarrow X \mid R \mathbin{;} S = S \mathbin{;} R \bullet iter\ a(R \mathbin{;} S) = iter\ a\ R \mathbin{;} iter\ a\ S$$

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