section setlaws

/Reference manual/Extended toolkit

1. Some Tactics and Laws for the CADiZ Theorem-prover

section *setlaws* parents *setdefs*, *corelaws*

This section contains general-purpose tactics, and laws, with (some of) their proofs, on the assumption that the definitions of setdefs.z are present.

Written by Sam Valentine.

Last updated September 1999.

The following tactics are supplied as part of the CADiZ system. They are intended to serve two purposes: a) to be directly useful in proof work; b) to be used as models when users develop their own tactics.

The current version of this library should be regarded as very provisional, since both the rules of inference and the tactic language are still in the course of development.

The proof system manipulates sequents. Each sequent used in a proof becomes a goal of the proof process, so in this document the words ”goal” and ”sequent” will be used interchangably. Tactics can take several sorts of argument. In this document we proceed ”top-down”, considering first those tactics which operate on goals, then those which operate on predicates, then those which operate on expressions, and finally those for schema texts and declarations.

The tactics given below are those which ”blow” a predicate, one or more expressions, a schema text, a series of declarations, etc. They make frequent recursive calls on themselves and on each other. Their design aim is to apply all simplifications which one would always want, except perhaps in very special circumstances, but to do nothing else. They are intended never to fail, but may cause ”Nothing changed” to be reported.

”blowPred” takes a single predicate argument, which may appear anywhere and be of any form. The main section of the tactic has a matcher which fans out into some eleven different cases, corresponding to the sort of predicate supplied. For each of these any applicable immediate simplifications are sought, usually after a recursive call to simplify the constituent elements. Finally, ”resolution” and ”linear decision” are tried, using the result of the previous simplifications. ”blowPred” always succeeds. If it achieves nothing, the report ”Nothing changed” is given.

$$
\begin{array}{l}
blowPred pred p \mid rec repeat \bullet \\
\quad match p :: \\
\quad expr q, r \mid q = r \bullet \\
\qquad \text{“apply tactic”}\ q\ \text{“blowExpr”};\ \text{“apply tactic”}\ r\ \text{“blowExpr”}; \\
\qquad !(\ \text{“absorption”}\ p \lor \\
\qquad match p :: \\
\qquad\quad \mid (\_exprs) = (\_exprs) \bullet \\
\qquad\quad \text{“expansion”}\ p;\ repeat :: \\
\qquad\quad \mid \langle\!|\ \_decls\ |\!\rangle = \langle\!|\ \_decls\ |\!\rangle \bullet \\
\qquad\quad \text{“expansion”}\ p;\ repeat :: \\
\qquad\quad \mid p \bullet skip :: .) :: \\
\quad expr q, r \mid q \in r \bullet \\
\qquad match r :: \\
\qquad\quad \mid (\_ \notin \_) \bullet !(\ \text{“expansion”}\ r \lor skip) :: \\
\qquad\quad \mid (\_ \neq \_) \bullet !(\ \text{“expansion”}\ r \lor skip) :: \\
\qquad\quad \mid id\ \_expr \bullet !(\ \text{“expansion”}\ r \lor skip) :: \\
\qquad\quad \mid r \bullet skip :: .\ ; \\
\qquad \text{“apply tactic”}\ q\ \text{“blowExpr”};\ \text{“apply tactic”}\ r\ \text{“blowExpr”}; \\
\qquad !(\ \text{“absorption”}\ p;\ repeat \lor \text{“expansion”}\ p;\ repeat \lor skip) :: \\
\quad stxt dec;\ pred prred \\
\qquad\quad \mid \exists\ dec \bullet prred \bullet \\
\qquad\quad \mid \forall\ dec \bullet prred \bullet \\
\qquad \text{“apply tactic”}\ dec\ \text{“blowStxt”}; \\
\qquad \text{“apply tactic”}\ prred\ \text{“blowPred”}; \\
\qquad !(\ \text{“absorption”}\ p;\ repeat \lor \text{“one-point”}\ p;\ repeat \lor skip) :: \\
\quad stxt dec;\ pred prred \mid \exists_1\ dec \bullet prred \bullet \\
\qquad \text{“apply tactic”}\ dec\ \text{“blowStxt”}; \\
\qquad \text{“apply tactic”}\ prred\ \text{“blowPred”}; \\
\qquad !(\ \text{“absorption”}\ p \lor \text{“one-point”}\ p \lor \text{“expansion”}\ p);\ repeat ::
\end{array}
$$

”blowExpr” is the expression counterpart of ”blowPred”. It takes a single expression argument, which may appear anywhere and be of any form. The main section of the tactic has a matcher which fans out into over twenty different cases, corresponding to the sort of expression supplied. For each of these any applicable immediate simplifications are sought, usually after a recursive call to simplify the constituent elements. ”blowExpr” always succeeds. If it achieves nothing, the report ”Nothing changed” is given.

$$
\begin{array}{l}
\mathit{blowExprexprt} \mid \mathit{recrepeat} \bullet \mathit{matcht} :: \\
\quad \mid \theta\_\mathit{expr} \bullet \text{“}\mathit{expansion}\text{”}\ t :: \\
\quad \mathit{exprp} \mid p.1 \bullet \\
\qquad\qquad \mid p.2 \bullet \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ p\ \ \text{“}\mathit{blowExpr}\text{”};\ !(\text{“}\mathit{absorption}\text{”}\ t \lor \mathit{skip}) :: \\
\quad \mathit{exprses} \mid (es) \bullet \text{“}\mathit{apply\ tactic}\text{”}\ es\ \ \text{“}\mathit{blowExprs}\text{”} :: \\
\quad \mathit{declsds} \mid \langle\!|\ ds\ |\!\rangle \bullet \text{“}\mathit{apply\ tactic}\text{”}\ ds\ \ \text{“}\mathit{blowConstDecls}\text{”} :: \\
\quad \mathit{expre} \mid \mathbb{P}\, e \bullet \text{“}\mathit{apply\ tactic}\text{”}\ e\ \ \text{“}\mathit{blowExpr}\text{”} :: \\
\quad \mathit{exprp}, q \mid p \times q \bullet \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ p\ \ \text{“}\mathit{blowExpr}\text{”};\ \ \text{“}\mathit{apply\ tactic}\text{”}\ q\ \ \text{“}\mathit{blowExpr}\text{”} :: \\
\quad \mathit{predp};\ \mathit{exprq}, r \mid \mathrm{if}\ p\ \mathrm{then}\ q\ \mathrm{else}\ r \bullet \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ p\ \ \text{“}\mathit{blowPred}\text{”}; \\
\qquad\quad (\text{“}\mathit{absorption}\text{”}\ t;\ \mathit{repeat} \lor \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ q\ \ \text{“}\mathit{blowExpr}\text{”};\ \ \text{“}\mathit{apply\ tactic}\text{”}\ r\ \ \text{“}\mathit{blowExpr}\text{”}; \\
\qquad\quad (\text{“}\mathit{absorption}\text{”}\ t \lor \mathit{skip})) :: \\
\quad \mathit{stxts};\ \mathit{exprct} \mid \{s \bullet ct\} \bullet \\
\qquad\qquad \mid \lambda\, s \bullet ct \bullet \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ s\ \ \text{“}\mathit{blowStxt}\text{”}; \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ ct\ \ \text{“}\mathit{blowExpr}\text{”}; \\
\qquad\quad !(\text{“}\mathit{absorption}\text{”}\ t \lor \text{“}\mathit{one\text{-}point}\text{”}\ t;\ \mathit{repeat} \lor \mathit{skip}) :: \\
\quad \mathit{stxts};\ \mathit{exprct} \mid \mu\, s \bullet ct \bullet \\
\qquad\qquad \mid \mathrm{let}\ s \bullet ct \bullet \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ s\ \ \text{“}\mathit{blowStxt}\text{”}; \\
\qquad\quad \text{“}\mathit{apply\ tactic}\text{”}\ ct\ \ \text{“}\mathit{blowExpr}\text{”}; \\
\qquad\quad !(\text{“}\mathit{absorption}\text{”}\ t;\ \mathit{repeat} \lor \text{“}\mathit{one\text{-}point}\text{”}\ t;\ \mathit{repeat} \lor \mathit{skip}) :: \\
\quad \mathit{exprfun}, \mathit{args} \mid \mathit{fun}\ \ \mathit{args} \bullet \\
\qquad\quad \mathit{matchfun} :: \\
\qquad\qquad \mid (\_ \mapsto \_) \bullet !(\text{“}\mathit{expansion}\text{”}\ \mathit{fun} \lor \mathit{skip}) :: \\
\qquad\qquad \mid \mathit{first} \bullet !(\text{“}\mathit{expansion}\text{”}\ \mathit{fun} \lor \mathit{skip}) ::
\end{array}
$$

$$
\begin{array}{l}
blowExprsexprsts \mid matchts :: expre;\ \ exprses \mid e, es \bullet \\
\quad \text{“}apply\ tactic\text{”}\ e\ \ \text{“}blowExpr\text{”}; \\
\quad \text{“}apply\ tactic\text{”}\ es\ \ \text{“}blowExprs\text{”} ::\mid ts \bullet skip :: .
\end{array}
$$

$$
\begin{array}{l}
blowStxtstxts \mid \\
\quad matchs \\
\quad :: declsds \mid ds \mid \_pred \bullet \\
\quad \text{“}apply\ tactic\text{”}\ ds\ \ s\ \ \text{“}blowDecls\text{”}; \\
\quad matchs :: predbarpart2 \mid \_decls \mid barpart2 \bullet \\
\quad \text{“}apply\ tactic\text{”}\ barpart2\ \ \text{“}blowPred\text{”} \\
\quad :: . \\
\quad :: .
\end{array}
$$

$blowDeclsdeclsds; \; stxts \mid$

$matchds$

$:: decld; \; declsds2 \mid d; \; ds2 \bullet$

$matchd$

$:: expre \mid \_name : e \bullet$

$``apply\ tactic" \; e \; ``blowExpr"$

$; \; matche$

$::| \; \{\_stxt \bullet \_expr\} \bullet ``normalization" \; d$

$::| \; \{\_exprs\} \bullet ``normalization" \; d$

$::| \; e \bullet skip$

$:: \, .$

$:: expre \mid \_name == e \bullet ``apply\ tactic" \; e \; ``blowExpr"$

$:: expre \mid e \bullet$

$``apply\ tactic" \; e \; ``blowExpr";$

$!(``distribution" \; d \lor skip)$

$:: \, . \; ;$

$matchds$

$:: decldd; \; declsdds2 \mid dd; \; dds2 \bullet$

$``apply\ tactic" \; dds2 \; s \; ``blowDecls"$

$:: \, .$

$::| \bullet skip$

$:: \, .$

$$
\begin{array}{l}
\mathit{blowConstDeclsdeclsds} \mid \\
\quad \mathit{matchds} \\
\quad :: \mathit{expre};\ \ \mathit{declsds2} \mid \_\mathit{name} == e;\ \ \mathit{ds2} \bullet \\
\qquad \text{“}\mathit{apply\ tactic}\text{”}\ e\ \ \text{“}\mathit{blowExpr}\text{”};\ \ \text{“}\mathit{apply\ tactic}\text{”}\ \mathit{ds2}\ \ \text{“}\mathit{blowConstDecls}\text{”} \\
\quad ::| \bullet\ \mathit{skip} \\
\quad ::\ .
\end{array}
$$

2. Sets

neqCommutes ==

$$[X] \vdash ?\ \forall x, y : X \mid x \neq y \bullet y \neq x$$

inNull ==

$$[X] \vdash ?\ \forall x : X \bullet x \notin \varnothing$$

subDef ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \subseteq T \Leftrightarrow S \in \mathbb{P}\, T$$

subSelf ==

$$[X] \vdash ?\ \forall S : \mathbb{P}\, X \bullet S \subseteq S$$

psubSelf ==

$$[X] \vdash ?\ \forall S : \mathbb{P}\, X \bullet \neg\, (S \subset S)$$

subsetSymEq ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \subseteq T \land T \subseteq S \Leftrightarrow S = T$$

L7 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet \neg\, (S \subset T \land T \subset S)$$

L8 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P}\, X \mid S \subseteq T \land T \subseteq V \bullet S \subseteq V$$

L9 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P}\, X \mid S \subset T \land T \subset V \bullet S \subset V$$

L10 ==

$$[X] \vdash ?\ \forall S : \mathbb{P}\, X \bullet \varnothing \subseteq S$$

L11 ==

$$[X] \vdash ?\ \forall S : \mathbb{P}\, X \bullet \varnothing \subset S \Leftrightarrow S \neq \varnothing$$

L12 ==

$$[Y] \vdash ?\ \forall S : \mathbb{P}\, Y \bullet \mathbb{P}_1\, S = \varnothing \Leftrightarrow S = \varnothing$$

L13 ==

$$[Y] \vdash ?\ \forall S : \mathbb{P}\, Y \bullet S \neq \varnothing \Leftrightarrow S \in \mathbb{P}_1\, S$$

L14 ==

$$[X] \vdash ?\ \forall S : \mathbb{P}\, X \bullet S \cup S = S \cup \varnothing = S \cap S = S \ominus \varnothing = S \setminus \varnothing = S$$

L15 ==

$$[X] \vdash ?\ \forall S : \mathbb{P}\, X \bullet S \cap \varnothing = S \ominus S = S \setminus S = \varnothing \setminus S = \varnothing$$

L16 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \cup T = T \cup S$$

L17 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \cap T = T \cap S$$

symdiffCommutes ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \ominus T = T \ominus S$$

L18 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P}\, X \bullet S \cup (T \cup V) = (S \cup T) \cup V$$

L19 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet S \cap (T \cap V) = (S \cap T) \cap V$$

symmdiffAssoc ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet S \ominus (T \ominus V) = (S \ominus T) \ominus V$$

L20 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet S \cup (T \cap V) = (S \cup T) \cap (S \cup V)$$

L21 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet S \cap (T \cup V) = (S \cap T) \cup (S \cap V)$$

intThruSymdiff ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet S \cap (T \ominus V) = (S \cap T) \ominus (S \cap V)$$

setsubThruSymdiff ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet (S \ominus T) \setminus V = (S \setminus V) \ominus (T \setminus V)$$

L24 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet S \setminus (T \setminus V) = (S \setminus T) \cup (S \cap V)$$

L25 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P} X \bullet (S \setminus T) \setminus V = S \setminus (T \cup V)$$

L26 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P}\, X \bullet S \cup (T \setminus V) = (S \cup T) \setminus (V \setminus S)$$

L27 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P}\, X \bullet S \cap (T \setminus V) = (S \cap T) \setminus V$$

L28 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P}\, X \bullet (S \cup T) \setminus V = (S \setminus V) \cup (T \setminus V)$$

L29 ==

$$[X] \vdash ?\ \forall S, T, V : \mathbb{P}\, X \bullet S \setminus (T \cap V) = (S \setminus T) \cup (S \setminus V)$$

subsetUnion ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \subseteq T \Leftrightarrow S \cup T = T$$

subsetInt ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \subseteq T \Leftrightarrow S \cap T = S$$

subsetSetminus ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \subseteq T \Leftrightarrow S \setminus T = \varnothing$$

powersetInt ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet \mathbb{P}(S \cap T) = \mathbb{P}\, S \cap \mathbb{P}\, T$$

L43 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \subseteq S \cup T$$

L44 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet T \subseteq S \cup T$$

L45 ==

$$[X] \vdash ?\ \forall S, T, W : \mathbb{P}\, X \mid S \subseteq W \land T \subseteq W \bullet S \cup T \subseteq W$$

L48 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \cap T \subseteq S$$

L49 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \cap T \subseteq T$$

L50 ==

$$[X] \vdash ?\ \forall S, T, W : \mathbb{P}\, X \mid W \subseteq S \land W \subseteq T \bullet W \subseteq S \cap T$$

L53 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet S \setminus T \subseteq S$$

L54 ==

$$[X] \vdash ?\ \forall S, T : \mathbb{P}\, X \bullet (S \setminus T) \cap T = \varnothing$$

L55 ==

$$[X] \vdash ?\ \forall S, T, W : \mathbb{P}\, X \mid W \subseteq S \wedge W \cap T = \varnothing \bullet W \subseteq S \setminus T$$

L30 ==

$$[X] \vdash ?\ \forall A, B : \mathbb{P}(\mathbb{P}\, X) \bullet \bigcup(A \cup B) = (\bigcup A) \cup (\bigcup B)$$

L31 ==

$$[X] \vdash ?\ \forall A, B : \mathbb{P}(\mathbb{P}\, X) \bullet \bigcap(A \cup B) = (\bigcap A) \cap (\bigcap B)$$

L32 ==

$$[X] \vdash ?\ \bigcup[X]\varnothing = \varnothing$$

L33 ==

$$[X] \vdash ?\ \bigcap[X]\varnothing = X$$

L34 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet S \cap (\bigcup A) = \bigcup\{T : A \bullet S \cap T\}$$

L35 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet S \cup (\bigcap A) = \bigcap\{T : A \bullet S \cup T\}$$

CADiZ

Page 15 of 35

Go Back

Full Screen

Close

Quit

L36 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet (\bigcup A) \setminus S = \bigcup \{ T : A \bullet T \setminus S \}$$

L37 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet S \setminus (\bigcap A) = \bigcup \{ T : A \bullet S \setminus T \}$$

cupBigcup ==

$$[X] \vdash ?\ \forall A : \mathbb{P}_1\, \mathbb{P}\, X;\ S : \mathbb{P}\, X \bullet S \cup (\bigcup A) = \bigcup \{ T : A \bullet S \cup T \}$$

capBigcap ==

$$[X] \vdash ?\ \forall A : \mathbb{P}_1(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet S \cap (\bigcap A) = \bigcap \{ T : A \bullet S \cap T \}$$

bigcupCup ==

$$[X] \vdash ?\ \forall A : \mathbb{P}_1\, \mathbb{P}\, X;\ S : \mathbb{P}\, X \bullet (\bigcup A) \cup S = \bigcup \{ T : A \bullet T \cup S \}$$

bigcapCap ==

$$[X] \vdash ?\ \forall A : \mathbb{P}_1(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet (\bigcap A) \cap S = \bigcap \{ T : A \bullet T \cap S \}$$

L38 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}_1(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet S \setminus (\bigcup A) = \bigcap \{ T : A \bullet S \setminus T \}$$

L39 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}_1(\mathbb{P}\, X);\ S : \mathbb{P}\, X \bullet (\bigcap A) \setminus S = \bigcap \{ T : A \bullet T \setminus S \}$$

L40 ==

$$[X] \vdash ?\ \forall A, B : \mathbb{P}(\mathbb{P}\, X) \mid A \subseteq B \bullet \bigcup A \subseteq \bigcup B$$

L41 ==

$$[X] \vdash ?\ \forall A, B : \mathbb{P}(\mathbb{P}\, X) \mid A \subseteq B \bullet \bigcap B \subseteq \bigcap A$$

L46 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X) \bullet \forall S : A \bullet S \subseteq \bigcup A$$

L47 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X);\ W : \mathbb{P}\, X \mid (\forall S : A \bullet S \subseteq W) \bullet \bigcup A \subseteq W$$

L51 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X) \bullet \forall S : A \bullet \bigcap A \subseteq S$$

L52 ==

$$[X] \vdash ?\ \forall A : \mathbb{P}(\mathbb{P}\, X);\ W : \mathbb{P}\, X \mid (\forall S : A \bullet W \subseteq S) \bullet W \subseteq \bigcap A$$

olddef ==

$$[X] \vdash ?\ \mathbb{F}_1\, X = \bigcap \{ A : \mathbb{P}(\mathbb{P}\, X) \mid \forall\, x : X \bullet \{x\} \in A \land \forall\, a : A \bullet a \cup \{x\} \in A \}$$

finiteNonEmptySets1 ==

$$[X] \vdash ?\ \forall\, S : \mathbb{P}\, X \bullet \mathbb{F}_1\, S = \mathbb{F}\, S \setminus \{\varnothing\}$$

finiteNonEmptySets2 ==

$$[X] \vdash ?\ \forall\, S : \mathbb{P}\, X \bullet \mathbb{F}\, S = \mathbb{F}_1\, S \cup \{\varnothing\}$$

auxfiniteIntersection ==

$$[X] \vdash ?\ \forall\, S : \mathbb{P}\, X \bullet \mathbb{F}\, S = \mathbb{F}\, X \cap \mathbb{P}\, S$$

finiteIntersection ==

$$[X] \vdash ?\ \forall\, S, T : \mathbb{P}\, X \bullet \mathbb{F}(S \cap T) = \mathbb{F}\, S \cap \mathbb{P}\, T$$

3. Proof by induction

Mathematical induction provides a method of proving properties of all members of finite sets. The induction principles are formulated as follows.

simpleFiniteSetInduction ==

$$[X] \vdash ?\ \forall P : \mathbb{P}\,\mathbb{P}\,X \mid \varnothing \in P \land \forall S : P;\ x : X \bullet S \cup \{x\} \in P \bullet \mathbb{F}\,X \subseteq P$$

simpleNonemptyFiniteSetInduction ==

$$[X] \vdash ?\ \forall P : \mathbb{P}\,\mathbb{P}\,X \mid \forall x : X \bullet \{x\} \in P \land \forall S : P \bullet S \cup \{x\} \in P \bullet \mathbb{F}_1\,X \subseteq P$$

cumulativeFiniteSetInduction ==

$$[X] \vdash ?\ \forall P : \mathbb{P}\,\mathbb{P}\,X \mid \forall S : \mathbb{P}\,X \mid \forall T : \mathbb{P}\,S \mid T \neq S \bullet T \in P \bullet S \in P \bullet \mathbb{F}\,X \subseteq P$$

cumulativeNonemptyFiniteSetInduction ==

$$[X] \vdash ?\ \forall P : \mathbb{P}\,\mathbb{P}\,X \mid \forall S : \mathbb{P}\,X \mid \forall T : \mathbb{P}_1\,S \mid T \neq S \bullet T \in P \bullet S \in P \bullet \mathbb{F}_1\,X \subseteq P$$

4. Relations

L42 ==

$$[X, Y] \vdash ?\ \forall p : X \times Y \bullet (\mathit{first}\ p, \mathit{second}\ p) = p$$

L56 ==

$$[X, Y] \vdash ?\ \forall x : X;\ R : X \leftrightarrow Y \bullet x \in dom\, R \Leftrightarrow (\exists y : Y \bullet x \mapsto y \in R)$$

L57 ==

$$[X, Y] \vdash ?\ \forall y : Y;\ R : X \leftrightarrow Y \bullet y \in ran\, R \Leftrightarrow (\exists x : X \bullet x \mapsto y \in R)$$

L58 ==

$$[X, Y] \vdash ?\ \forall S, T : X \leftrightarrow Y \bullet dom(S \cup T) = (dom\ S) \cup (dom\ T)$$

L59 ==

$$[X, Y] \vdash ?\ \forall S, T : X \leftrightarrow Y \bullet ran(S \cup T) = (ran\ S) \cup (ran\ T)$$

L60 ==

$$[X, Y] \vdash ?\ \forall S, T : X \leftrightarrow Y \bullet dom(S \cap T) \subseteq (dom\ S) \cap (dom\ T)$$

L61 ==

$$[X, Y] \vdash ?\ \forall S, T : X \leftrightarrow Y \bullet ran(S \cap T) \subseteq (ran\ S) \cap (ran\ T)$$

L62 ==

$$[X, Y] \vdash ?\ \ dom[X, Y]\varnothing = \varnothing$$

L63 ==

$$[X, Y] \vdash ?\ \ ran[X, Y]\varnothing = \varnothing$$

L64 ==

$$[X] \vdash ?\ \forall x, x' : X;\ s : \mathbb{P}\, X \bullet x \mapsto x' \in id\ s \Leftrightarrow x = x' \land x \in s$$

L65 ==

$$[X, Y, Z] \vdash ?\ \forall x : X;\ z : Z;\ R : X \leftrightarrow Y;\ S : Y \leftrightarrow Z \bullet \\ x \mapsto z \in R \mathbin{⨾} S \Leftrightarrow (\exists y : Y \bullet x \mapsto y \in R \land y \mapsto z \in S)$$

L66 ==

$$[W, X, Y, Z] \vdash ?\ \forall R : W \leftrightarrow X;\ S : X \leftrightarrow Y;\ T : Y \leftrightarrow Z \bullet R \mathbin{⨾} (S \mathbin{⨾} T) = (R \mathbin{⨾} S) \mathbin{⨾}$$

L67 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet id\ X \mathbin{⨾} R = R$$

L68 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet R \mathbin{⨾} id\ Y = R$$

L69 ==

$$[X] \vdash ?\ \forall V, W : \mathbb{P}\, X \bullet id\ V \mathbin{⨾} id\ W = id(V \cap W)$$

rightComposeThruUnion ==

$$[X, Y, Z] \vdash ?\ \forall R, S : X \leftrightarrow Y;\ T : Y \leftrightarrow Z \bullet (R \cup S) \mathbin{;} T = (R \mathbin{;} T) \cup (S \mathbin{;} T)$$

leftComposeThruUnion ==

$$[X, Y, Z] \vdash ?\ \forall R : X \leftrightarrow Y;\ S, T : Y \leftrightarrow Z \bullet R \mathbin{;} (S \cup T) = (R \mathbin{;} S) \cup (R \mathbin{;} T)$$

L71 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P} X \bullet S \lhd R = \mathit{id}\, S \mathbin{;} R = (S \times Y) \cap R$$

L72 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ T : \mathbb{P} Y \bullet R \rhd T = R \mathbin{;} \mathit{id}\, T = R \cap (X \times T)$$

L73 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P} X \bullet \mathit{dom}(S \lhd R) = S \cap (\mathit{dom}\, R)$$

L74 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ T : \mathbb{P} Y \bullet \mathit{ran}(R \rhd T) = (\mathit{ran}\, R) \cap T$$

L75 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P} X \bullet S \lhd R \subseteq R$$

Go Back

Full Screen

Close

Quit

L76 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ T : \mathbb{P}\, Y \bullet R \rhd T \subseteq R$$

L77 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P}\, X;\ T : \mathbb{P}\, Y \bullet (S \lhd R) \rhd T = S \lhd (R \rhd T)$$

L78 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S, V : \mathbb{P}\, X \bullet S \lhd (V \lhd R) = (S \cap V) \lhd R$$

L79 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ T, W : \mathbb{P}\, Y \bullet (R \rhd T) \rhd W = R \rhd (T \cap W)$$

L80 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P}\, X \bullet S \ndres R = (X \setminus S) \lhd R$$

L81 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ T : \mathbb{P}\, Y \bullet R \nrres T = R \rhd (Y \setminus T)$$

L82 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P}\, X \bullet (S \lhd R) \cup (S \ndres R) = R$$

L83 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ T : \mathbb{P}\, Y \bullet (R \rhd T) \cup (R \nrres T) = R$$

L77a ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P}\, X;\ T : \mathbb{P}\, Y \bullet (S \ndres R) \nrres T = S \ndres (R \nrres T)$$

L77b ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P}\, X;\ T : \mathbb{P}\, Y \bullet (S \ndres R) \rhd T = S \ndres (R \rhd T)$$

L77c ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P}\, X;\ T : \mathbb{P}\, Y \bullet (S \lhd R) \nrres T = S \lhd (R \nrres T)$$

L84 ==

$$[X, Y] \vdash ?\ \forall x : X;\ y : Y;\ R : X \leftrightarrow Y \bullet y \mapsto x \in R^{\sim} \Leftrightarrow x \mapsto y \in R$$

L85 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet (R^{\sim})^{\sim} = R$$

L86 ==

$$[X, Y, Z] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : Y \leftrightarrow Z \bullet (R \comp S)^{\sim} = S^{\sim} \comp R^{\sim}$$

L87 ==

$[V] \vdash ?\ (id\ V)^{\sim} = id\ V$

L88 ==

$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet dom(R^{\sim}) = ran\ R$

L89 ==

$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet ran(R^{\sim}) = dom\ R$

L90 ==

$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet id(dom\ R) \subseteq R \fatsemi R^{\sim}$

L91 ==

$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet id(ran\ R) \subseteq R^{\sim} \fatsemi R$

L92 ==

$[X, Y] \vdash ?\ \forall y : Y;\ R : X \leftrightarrow Y;\ S : \mathbb{P}\, X \bullet y \in R(\!|\ S\ |\!) \Leftrightarrow (\exists x : X \bullet x \in S \land x \mapsto$

L93 ==

$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S : \mathbb{P}\, X \bullet R(\!|\ S\ |\!) = ran(S \lhd R)$

L94 ==

$$[X, Y, Z] \vdash ?\ \forall Q : X \leftrightarrow Y;\ R : Y \leftrightarrow Z \bullet dom(Q \mathbin{;} R) = Q^{\sim}(\!|\ dom\ R\ |\!)$$

L95 ==

$$[X, Y, Z] \vdash ?\ \forall Q : X \leftrightarrow Y;\ R : Y \leftrightarrow Z \bullet ran(Q \mathbin{;} R) = R(\!|\ ran\ Q\ |\!)$$

L96 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S, T : \mathbb{P}\, X \bullet R(\!|\ S \cup T\ |\!) = R(\!|\ S\ |\!) \cup R(\!|\ T\ |\!)$$

L97 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S, T : \mathbb{P}\, X \bullet R(\!|\ S \cap T\ |\!) \subseteq R(\!|\ S\ |\!) \cap R(\!|\ T\ |\!)$$

L98 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet R(\!|\ dom\ R\ |\!) = ran\ R$$

L99 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet dom\ R = first(\!|\ R\ |\!)$$

L100 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet ran\ R = second(\!|\ R\ |\!)$$

bound1 ==

$$[X, Y] \vdash ?\ \forall y : Y;\ R : X \leftrightarrow Y;\ S : \mathbb{P}\, X \bullet \\ y \in upperBound\ R\ S \Leftrightarrow (\forall x : S \bullet x \mapsto y \in R)$$

bound5 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S, T : \mathbb{P}\, X \bullet \\ (upperBound\ R\ S) \cup (upperBound\ R\ T) \subseteq upperBound\ R\ (S \cap T)$$

bound6 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ S, T : \mathbb{P}\, X \bullet \\ (upperBound\ R\ S) \cap (upperBound\ R\ T) = upperBound\ R\ (S \cup T)$$

bound7 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ A, S : \mathbb{P}\, X;\ T : \mathbb{P}\, Y \mid A \subseteq S \bullet \\ T \setminus (upperBound\ R\ A) = ((S \times T) \setminus R)(\!|\ A\ |\!)$$

bound8 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y;\ A, S : \mathbb{P}\, X;\ T : \mathbb{P}\, Y \bullet \\ ((S \times T) \setminus R)(\!|\ A\ |\!) = T \setminus (upperBound\ R\ (S \cap A))$$

L198 ==

$$[I, X] \vdash ?\ \ disjoint\, \varnothing[I \times \mathbb{P}\, X]$$

otherPartition ==

$$\begin{array}{l}[I, X] \vdash ?\ (\_\, partition\, \_) = \\ \quad \{S : I \leftrightarrow \mathbb{P}\, X;\ T : \mathbb{P}\, X \mid \\ \quad\quad (\forall x : T \bullet \exists_1 p : S \bullet x \in p.2) \land (\forall p : S \bullet \forall x : p.2 \bullet x \in T)\}\end{array}$$

otherOverride ==

$$\begin{array}{l}[X, Y] \vdash ?\ (\_ \oplus \_) = \lambda\, Q, R : X \leftrightarrow Y \bullet \\ \quad \{p : X \times Y \mid p \in R \lor p \in Q \land \lnot \exists y : Y \bullet (p.1, y) \in R\}\end{array}$$

overrideClosed ==

$$[X, Y] \vdash ?\ \forall Q, R : X \leftrightarrow Y \bullet Q \oplus R \in X \leftrightarrow Y$$

L101 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet R \oplus R = R$$

L102 ==

$$[X, Y] \vdash ?\ \forall P, Q, R : X \leftrightarrow Y \bullet P \oplus (Q \oplus R) = (P \oplus Q) \oplus R$$

L103 ==

$$[X, Y] \vdash ?\ \forall R : X \leftrightarrow Y \bullet \varnothing \oplus R = R \oplus \varnothing = R$$

L104 ==

$$[X, Y] \vdash ?\ \forall Q, R : X \leftrightarrow Y \bullet dom(Q \oplus R) = (dom\ Q) \cup (dom\ R)$$

L105 ==

$$[X, Y] \vdash ?\ \forall Q, R : X \leftrightarrow Y \mid dom\ Q \cap dom\ R = \varnothing \bullet Q \oplus R = Q \cup R$$

L106 ==

$$[X, Y] \vdash ?\ \forall V : \mathbb{P}\, X;\ Q, R : X \leftrightarrow Y \bullet V \lhd (Q \oplus R) = (V \lhd Q) \oplus (V \lhd R)$$

L107 ==

$$[X, Y] \vdash ?\ \forall Q, R : X \leftrightarrow Y;\ W : \mathbb{P}\, Y \bullet (Q \oplus R) \rhd W \subseteq (Q \rhd W) \oplus (R \rhd W)$$

5. Orders

chainExample ==

$$[X] \vdash ?\ \forall a, b, c : X \bullet \{\{a, b, c\}, \{a, b\}, \{a\}\} \in irreflexiveChain(\_ \subseteq \_)[X]$$

L110 ==

$$[X] \vdash ? \ \forall R : X \leftrightarrow X \bullet R \subseteq R^{+}$$

L111 ==

$$[X] \vdash ? \ \forall R : X \leftrightarrow X \bullet R^{+} \mathbin{⨾} R^{+} \subseteq R^{+}$$

L112 ==

$$[X] \vdash ? \ \forall R, Q : X \leftrightarrow X \mid R \subseteq Q \land R \mathbin{⨾} Q \subseteq Q \bullet R^{+} \subseteq Q$$

L113 ==

$$[X] \vdash ? \ \forall R : X \leftrightarrow X \bullet \mathit{id}\ X \subseteq R^{*}$$

L114 ==

$$[X] \vdash ? \ \forall R : X \leftrightarrow X \bullet R \subseteq R^{*}$$

L115 ==

$$[X] \vdash ? \ \forall R : X \leftrightarrow X \bullet R^{*} \mathbin{⨾} R^{*} = R^{*}$$

L116 ==

$$[X] \vdash ? \ \forall R, Q : X \leftrightarrow X \mid \mathit{id}\ X \subseteq Q \land R \mathbin{⨾} Q \subseteq Q \bullet R^{*} \subseteq Q$$

L117a ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet R^* = R^+ \cup id\ X$$

L117b ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet R^* = (R \cup id\ X)^+$$

L118a ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet R^+ = R \mathbin{⨾} R^*$$

L118b ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet R^+ = R^* \mathbin{⨾} R$$

L119 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet (R^+)^+ = R^+$$

L120 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet (R^*)^* = R^*$$

doAsClosure ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X \bullet do\ R = R^* \rhd dom\ R$$

doInduction ==

$$[X] \vdash ?\ \forall R, Q : X \leftrightarrow X \mid id(X \setminus dom\ R) \subseteq Q \land R \mathbin{⨾} Q \subseteq Q \bullet do\ R \subseteq Q$$

L121 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X;\ S : \mathbb{P}\, X \bullet S \subseteq R^{*}(\!|\ S\ |\!)$$

L122 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X;\ S : \mathbb{P}\, X \bullet R(\!|\ R^{*}(\!|\ S\ |\!)\ |\!) \subseteq R^{*}(\!|\ S\ |\!)$$

L123 ==

$$[X] \vdash ?\ \forall R : X \leftrightarrow X;\ S, T : \mathbb{P}\, X \mid S \subseteq T \land R(\!|\ T\ |\!) \subseteq T \bullet R^{*}(\!|\ S\ |\!) \subseteq T$$

6. Functions

L70 ==

$$[X, Y, Z] \vdash ?\ \forall f : Y \rightarrow Z;\ g : X \rightarrow Y;\ x : X \bullet (f \circ g) x = f(g(x))$$

L108 ==

$$[X, Y] \vdash ?\ \forall x : X;\ f, g : X \rightarrow Y \mid x \in (\mathrm{dom}\, f) \setminus (\mathrm{dom}\, g) \bullet (f \oplus g) x = f\ x$$

L109 ==

$$[X, Y] \vdash ?\ \forall x : X;\ f, g : X \rightarrow Y \mid x \in \mathrm{dom}\, g \bullet (f \oplus g) x = g\ x$$

L124 ==

$$[X, Y] \vdash ?\ \forall f : \mathbb{P}(X \times Y) \bullet f \in X \pfun Y \Leftrightarrow f \circ f^{\sim} = id(ran\, f)$$

L125 ==

$$[X, Y] \vdash ?\ \forall f : \mathbb{P}(X \times Y) \bullet f \in X \pinj Y \Leftrightarrow f \in X \pfun Y \land f^{\sim} \in Y \pfun X$$

L126 ==

$$[X, Y] \vdash ?\ \forall f : \mathbb{P}(X \times Y) \bullet f \in X \inj Y \Leftrightarrow f \in X \fun Y \land f^{\sim} \in Y \pfun X$$

L127 ==

$$[X, Y] \vdash ?\ \forall f : \mathbb{P}(X \times Y);\ S, T : \mathbb{P}\, X \mid f \in X \pinj Y \bullet f \limg S \cap T \rimg = f \limg S \rimg \cap f \limg T$$

L128 ==

$$[X, Y] \vdash ?\ \forall f : \mathbb{P}(X \times Y) \bullet f \in X \bij Y \Leftrightarrow f \in X \fun Y \land f^{\sim} \in Y \fun X$$

L129 ==

$$[X, Y] \vdash ?\ \forall f : \mathbb{P}(X \times Y) \mid f \in X \psurj Y \bullet f \circ f^{\sim} = id\ Y$$

L153 ==

$$[X, Y] \vdash ?\ X \ffun Y = (X \pfun Y) \cap \mathbb{F}(X \times Y)$$

Go Back

Full Screen

Close

Quit

auxPigeonhole ==

$$[X] \vdash ?\ \forall N == \{V : \mathbb{P}\, X \mid \forall U : \mathbb{P}\, V;\ R : X \leftrightarrow X \mid R \in U ⤀ V \bullet U = V\} \bullet \mathbb{F}\, X$$

Pigeonhole ==

$$[X] \vdash ?\ \forall S, T : \mathbb{F}\, X;\ f : X \leftrightarrow X \mid S \subseteq T \land f \in S ⤀ T \bullet S = T$$

L148 ==

$$[X] \vdash ?\ \forall S : \mathbb{F}\, X \bullet \forall f : S \rightarrowtail S \bullet \operatorname{ran} f = S$$

7. Relational operations on functions

F1 ==

$$[X] \vdash ?\ \forall S : \mathbb{P}\, X \bullet \mathit{id}\, S \in X ⤔ X$$

F2 ==

$$[X] \vdash ?\ \mathit{id}\, X \in X ⤖ X$$

F3 ==

$$[X, Y, Z] \vdash ?\ \forall f : X ⇸ Y;\ g : Y ⇸ Z \bullet g \circ f \in X ⇸ Z$$

F4 ==

$$[X, Y, Z] \vdash ?\ \forall f : X \rightarrow Y;\ g : Y \mathrel{⇸} Z \mid \mathit{ran}\, f \subseteq \mathit{dom}\, g \bullet g \circ f \in X \rightarrow Z$$

F5 ==

$$[X, Y] \vdash ?\ \forall S : \mathbb{P}\, X;\ f : X \mathrel{⇸} Y \bullet S \lhd f \in X \mathrel{⇸} Y$$

F6 ==

$$[X, Y] \vdash ?\ \forall T : \mathbb{P}\, Y;\ f : X \mathrel{⇸} Y \bullet f \rhd T \in X \mathrel{⇸} Y$$

F7 ==

$$[X, Y] \vdash ?\ \forall f : X \mathrel{⇸} Y;\ g : X \mathrel{⇸} Y \bullet f \oplus g \in X \mathrel{⇸} Y$$

F8 ==

$$[X, Y, Z] \vdash ?\ \forall f : X \mathrel{⤔} Y;\ g : Y \mathrel{⤔} Z \bullet g \circ f \in X \mathrel{⤔} Z$$

F9 ==

$$[X, Y] \vdash ?\ \forall S : \mathbb{P}\, X;\ f : X \mathrel{⤔} Y \bullet S \lhd f \in X \mathrel{⤔} Y$$

F10 ==

$$[X, Y] \vdash ?\ \forall T : \mathbb{P}\, Y;\ f : X \mathrel{⤔} Y \bullet f \rhd T \in X \mathrel{⤔} Y$$

F11 ==

$$[X, Y] \vdash ?\ \forall f : X ⤔ Y \bullet f^{\sim} \in Y ⤔ X$$

F12 ==

$$[X, Y] \vdash ?\ \forall f : X ⇸ Y;\ g : X ⇸ Y \mid (\mathrm{dom}\, f) \lhd g = (\mathrm{dom}\, g) \lhd f \bullet f \cup g \in X ⇸$$

F13 ==

$$[X, Y] \vdash ?\ \forall f : X ⇸ Y;\ g : X \leftrightarrow Y \bullet f \cap g \in X ⇸ Y$$

F14 ==

$$[X, Y] \vdash ?\ \forall f : X ⤔ Y;\ g : X \leftrightarrow Y \bullet f \cap g \in X ⤔ Y$$

F15 ==

$$[X, Y] \vdash ?\ \forall f : X ⇸ Y;\ g : \mathbb{P}(X \times Y) \mid g \subseteq f \bullet g \in X ⇸ Y$$

F16 ==

$$[X, Y] \vdash ?\ \forall f : X ⤔ Y;\ g : \mathbb{P}(X \times Y) \mid g \subseteq f \bullet g \in X ⤔ Y$$

IT 22-Jan-2002