

section numlaws

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1. Laws about Numbers in \mathbb{Z}

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section *numlaws* parents *numdefs*, *corelaws*

This section contains the CADiZ version of some laws about numbers in the mathematical toolkit of \mathbb{Z} .

1.1. Prelude-based Laws

Those laws which depend solely on the definitions in the prelude are given in the file *corelaws.z*, which is a parent of this one.

1.2. Theorems about the Natural Numbers

We could here state and prove various useful theorems about \mathbb{N} , both because of their usefulness, and to assist in proving the consistency of the definitions in `numdefs`.

The theorems would mainly be to the effect that \mathbb{N} is an Abelian monoid under $_ * _$, and $_ * _$ distributes through $_ + _$.

`zeroTimesBEqZero ==`

$$\vdash? \forall b : \mathbb{N} \bullet 0 * b = 0$$

`OneTimesBEqB ==`

$$\vdash? \forall b : \mathbb{N} \bullet 1 * b = b$$

`TimesCommutates ==`

$$\vdash? \forall a, b : \mathbb{N} \bullet a * b = b * a$$

`TimesDistributesThruPlus ==`

$$\vdash? \forall a, b, c : \mathbb{N} \bullet a * (b + c) = (a * b) + (a * c)$$

`TimesClosed ==`

$$\vdash? \forall a, b : \mathbb{N} \bullet a * b \in \mathbb{N}$$

TimesAssociates ==

$$\vdash? \forall a, b, c : \mathbb{N} \bullet (a * b) * c = a * (b * c)$$

TimesConstInjective ==

$$\vdash? \forall a, b : \mathbb{N}; c : \mathbb{N}_1 \mid a * c = b * c \bullet a = b$$

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