factorization

The *factorization* command extracts a formula that is a common factor in both sub-formulae. Some of these are inverses of *distribution* steps. Unusually, these steps are organized by the syntax of the result, not that of the original formula.

1. Contents of this page

1.1. Conjunctions

$$
\begin{array}{rcl}
p_1 \wedge p_2 \wedge (p_1 \wedge p_3) & \Longrightarrow & p_1 \wedge (p_2 \wedge p_3) \\
p_1 \wedge p_3 \wedge (p_2 \wedge p_3) & \Longrightarrow & p_1 \wedge p_2 \wedge p_3 \\
p_1 \wedge p_2 \vee p_1 \wedge p_3 & \Longrightarrow & p_1 \wedge (p_2 \vee p_3) \\
p_1 \wedge p_3 \vee p_2 \wedge p_3 & \Longrightarrow & (p_1 \vee p_2) \wedge p_3
\end{array}
$$

1.2. Disjunctions

$$
\begin{array}{rcl}
p_1 \vee p_2 \vee (p_1 \vee p_3) & \Longrightarrow & p_1 \vee (p_2 \vee p_3) \\
p_1 \vee p_3 \vee (p_2 \vee p_3) & \Longrightarrow & p_1 \vee p_2 \vee p_3 \\
(p_1 \vee p_2) \wedge (p_1 \vee p_3) & \Longrightarrow & p_1 \vee p_2 \wedge p_3 \\
(p_1 \vee p_3) \wedge (p_2 \vee p_3) & \Longrightarrow & p_1 \wedge p_2 \vee p_3
\end{array}
$$

1.3. Implications

$$
\begin{array}{rcl}
(p_1 \Rightarrow p_2) \wedge (p_1 \Rightarrow p_3) & \Longrightarrow & p_1 \Rightarrow p_2 \wedge p_3 \\
(p_1 \Rightarrow p_3) \wedge (p_2 \Rightarrow p_3) & \Longrightarrow & p_1 \vee p_2 \Rightarrow p_3 \\
p_1 \wedge p_2 \Leftrightarrow p_1 \wedge p_3 & \Longrightarrow & p_1 \Rightarrow (p_2 \Leftrightarrow p_3)
\end{array}
$$

1.4. Universal quantifications

$$
\begin{array}{rcl}
(\forall\, s \bullet p_1) \land (\forall\, s \bullet p_2) & \Longrightarrow & \forall\, s \bullet p_1 \land p_2 \\
(\forall\, s \bullet p_1) \lor (\forall\, s \bullet p_2) & \Longrightarrow & \forall\, s \bullet p_1 \lor p_2 \text{ where } s \text{ has only } == \text{ decs} \\
(\forall\, s \bullet p_1) \Rightarrow (\forall\, s \bullet p_2) & \Longrightarrow & \forall\, s \bullet p_1 \Rightarrow p_2 \text{ where } s \text{ has only } == \text{ decs}
\end{array}
$$

1.5. Existential quantifications

$$
\begin{array}{rcl}
(\exists\, s \bullet p_1) \land (\exists\, s \bullet p_2) & \Longrightarrow & \exists\, s \bullet p_1 \land p_2 \text{ where } s \text{ has only } == \text{ decs} \\
(\exists\, s \bullet p_1) \lor (\exists\, s \bullet p_2) & \Longrightarrow & \exists\, s \bullet p_1 \lor p_2 \\
(\exists\, s \bullet p_1) \Rightarrow (\exists\, s \bullet p_2) & \Longrightarrow & \exists\, s \bullet p_1 \Rightarrow p_2 \text{ where } s \text{ has only } == \text{ decs}
\end{array}
$$

1.6. Unique existential quantifications

$$
\begin{array}{rcl}
(\exists_1\, s \bullet p_1) \land (\exists_1\, s \bullet p_2) & \Longrightarrow & \exists_1\, s \bullet p_1 \land p_2 \text{ where } s \text{ has only } == \text{ decs} \\
(\exists_1\, s \bullet p_1) \lor (\exists_1\, s \bullet p_2) & \Longrightarrow & \exists_1\, s \bullet p_1 \lor p_2 \text{ where } s \text{ has only } == \text{ decs} \\
(\exists_1\, s \bullet p_1) \Rightarrow (\exists_1\, s \bullet p_2) & \Longrightarrow & \exists_1\, s \bullet p_1 \Rightarrow p_2 \text{ where } s \text{ has only } == \text{ decs}
\end{array}
$$

2. Tactic example

"factorization" p_4 p_5

This example applies the *factorization* command to predicates p_4 and p_5.

IT 5-Apr-2000