section function_toolkit

1. Functions

section *function_toolkit* parents *relation_toolkit*

1.1. Partial functions

generic 5 rightassoc(_ $\pfun$ _)

$$X \pfun Y == \{ f : X \rel Y \mid \forall p, q : f \mid p.1 = q.1 \bullet p.2 = q.2 \}$$

$X \pfun Y$ is the set of all partial functions from X to Y, that is, the set of all relations between X and Y such that each x in X is related to at most one y in Y. The terms “function” and “partial function” are synonymous.

1.2. Partial injections

generic 5 rightassoc(_ $\pinj$ _)

$$X \pinj Y == \{\, f : X \rel Y \mid \forall p, q : f \bullet p.1 = q.1 \Leftrightarrow p.2 = q.2 \,\}$$

$X \pinj Y$ is the set of partial injections from X to Y, that is, the set of all relations between X and Y such that each x in X is related to no more than one y in Y, and each y in Y is related to no more than one x in X. The terms “injection” and “partial injection” are synonymous.

1.3. Total injections

$$\text{generic } 5 \text{ rightassoc}(\_ \inj \_)$$

$$X \inj Y == (X \pinj Y) \cap (X \fun Y)$$

$X \inj Y$ is the set of total injections from X to Y, that is, the set of injections from X to Y that are also total functions from X to Y.

1.4. Partial surjections

$$\text{generic } 5 \text{ rightassoc}(\_ \psurj \_)$$

$$X \psurj Y == \{\, f : X \pfun Y \mid \mathit{ran}\, f = Y \,\}$$

$X \psurj Y$ is the set of partial surjections from X to Y, that is, the set of functions from X to Y whose range is equal to Y. The terms “surjection” and “partial surjection” are synonymous.

1.5. Total surjections

$$\text{generic } 5 \text{ rightassoc}(\_ \twoheadrightarrow \_)$$

$$X \twoheadrightarrow Y == (X \psurj Y) \cap (X \to Y)$$

$X \twoheadrightarrow Y$ is the set of total surjections from X to Y, that is, the set of surjections from X to Y that are also total functions from X to Y.

1.6. Bijections

$$\text{generic } 5 \text{ rightassoc}(\_ \bij \_)$$

$$X \bij Y == (X \twoheadrightarrow Y) \cap (X \rightarrowtail Y)$$

$X \bij Y$ is the set of bijections from X to Y, that is, the set of total surjections from X to Y that are also total injections from X to Y.

1.7. Finite functions

$$\text{generic } 5 \text{ rightassoc}(\_ \ffun \_)$$

$$X \ffun Y == (X \pfun Y) \cap \mathbb{F}(X \times Y)$$

The finite functions from X to Y are the functions from X to Y that are also finite sets.

1.8. Finite injections

$$\text{generic } 5 \text{ rightassoc}(\_ \mathbin{⤕} \_)$$

$$X \mathbin{⤕} Y == (X \mathbin{⇻} Y) \cap (X \mathbin{⤔} Y)$$

The finite injections from X to Y are the injections from X to Y that are also finite functions from X to Y.

1.9. Disjointness

$$\text{relation}(\mathit{disjoint}\ \_)$$

$$
\begin{array}{l}
[L, X] \\
\hline
\mathit{disjoint}\ \_ : \mathbb{P}(L \leftrightarrow \mathbb{P}\, X) \\
\hline
\forall f : L \leftrightarrow \mathbb{P}\, X \bullet \mathit{disjoint}\ f \Leftrightarrow (\forall p, q : f \mid p \neq q \bullet p.2 \cap q.2 = \varnothing)
\end{array}
$$

A labelled family of sets is disjoint precisely when any distinct pair yields sets with no members in common.

1.10. Partitions

$$\text{relation}(\_\ \mathit{partition}\ \_)$$

$$
\begin{array}{l}
[L, X] \\
\hline
\_\ partition\ \_ : (L \leftrightarrow \mathbb{P}\, X) \leftrightarrow \mathbb{P}\, X \\
\hline
\forall f : L \leftrightarrow \mathbb{P}\, X;\ a : \mathbb{P}\, X \bullet f\ partition\ a \Leftrightarrow disjoint\ f \land \bigcup(\mathrm{ran}\ f) = a
\end{array}
$$

A labelled family of sets f partitions a set a precisely when f is disjoint and the union of all the sets in f is a.

IT 5-Jan-2002