section bagkit

/Reference manual/Extended toolkit

section *bagkit* parents *toolkit*

1. Bags

1.0.0.1. Name

bag	–	Bags
$count$, $\sharp$	–	Multiplicity
$\otimes$	–	Bag scaling

1.0.0.2. Definition

generic(bag _)

$$\mathrm{bag}\, X == X \pfun \mathbb{N}_1$$

$$count[X] == \lambda\, B : \mathrm{bag}\, X \bullet (\lambda\, x : X \bullet 0) \oplus B$$

function 50 leftassoc(_♯_)

CADiZ

$$\_\sharp\_[X] == \lambda\, B : \mathrm{bag}\, X;\ x : X \bullet \mathit{count}\ \ B\ \ x$$

$$\text{function } 40 \text{ leftassoc}(\_\otimes\_)$$

$$\_\otimes\_[X] == \lambda\, n : \mathbb{N};\ B : \mathrm{bag}\, X \bullet (\lambda\, x : X \bullet n * (B \sharp x)) \rhd \{0\}$$

$$\text{function}(\llbracket, ,\rrbracket)$$

$$\llbracket, ,\rrbracket[X] == \lambda\, x : \mathit{seq}\, X \bullet \mathit{items}\ \ x$$

1.0.0.3. Description

1.0.0.4. Laws

scaleOldDef ==

$$[X] \vdash ?\ \forall\, n : \mathbb{N};\ B : \mathrm{bag}\, X;\ x : X \bullet (n \otimes B)\sharp x = n * (B \sharp x)$$

L202 ==

$$[X] \vdash ?\ \forall\, n : \mathbb{N};\ B : \mathrm{bag}\, X \bullet n \otimes \llbracket\rrbracket = 0 \otimes B = \llbracket\rrbracket$$

L203 ==

$$[X] \vdash ?\ \forall\, B : \mathrm{bag}\, X \bullet 1 \otimes B = B$$

L204 ==

$$[X] \vdash ?\ \forall\, m, n : \mathbb{N};\ B : \mathrm{bag}\, X \bullet (n * m) \otimes B = n \otimes (m \otimes B)$$

1.0.0.5. Name

inbag – Bag membership

$\sqsubseteq$ – Sub-bag relation

1.0.0.6. Definition

$$\mathrm{relation}(\_ \mathrm{inbag} \_)$$

$$\_ \mathrm{inbag} \_[X] == \{x : X;\ B : \mathrm{bag}\ X \mid x \in \mathit{dom}\ B\}$$

$$\mathrm{relation}(\_ \sqsubseteq \_)$$

$$\_ \sqsubseteq \_[X] == \{B, C : \mathrm{bag}\ X \mid \forall x : X \bullet B \sharp x \leq C \sharp x\}$$

1.0.0.7. Description

1.0.0.8. Laws

L207 ==

$$[X] \vdash ?\ \forall x : X;\ B : \mathrm{bag}\ X \bullet x\ \mathrm{inbag}\ B \Leftrightarrow B \sharp x > 0$$

L208 ==

$$[X] \vdash ?\ \forall B, C : \mathrm{bag}\ X \mid B \sqsubseteq C \bullet \mathit{dom}\ B \subseteq \mathit{dom}\ C$$

L209 ==

$$[X] \vdash ?\ \forall B : \mathrm{bag}\, X \bullet [\![\,]\!] \sqsubseteq B$$

L210 ==

$$[X] \vdash ?\ \forall B : \mathrm{bag}\, X \bullet B \sqsubseteq B$$

L211 ==

$$[X] \vdash ?\ \forall B, C : \mathrm{bag}\, X \mid B \sqsubseteq C \land C \sqsubseteq B \bullet B = C$$

L212 ==

$$[X] \vdash ?\ \forall B, C, D : \mathrm{bag}\, X \mid B \sqsubseteq C \land C \sqsubseteq D \bullet B \sqsubseteq D$$

1.0.0.9. Name

$\uplus$ – Bag sum
$\uminus$ – Bag difference

1.0.0.10. Definition

function 30 leftassoc$(_ \uplus _)$

$$\_ \uplus \_[X] == \lambda\, B, C : \mathrm{bag}\, X \bullet (\lambda\, x : X \bullet B \sharp x + C \sharp x) \rhd \{0\}$$

function 30 leftassoc$(_ \uminus _)$

$$\_ \uminus \_[X] == \lambda\, B, C : \mathrm{bag}\, X \bullet (\lambda\, x : X \bullet max\{B \sharp x - C \sharp x, 0\}) \rhd \{0\}$$

1.0.0.11. Description

1.0.0.12. Laws

L213 ==

$$[X] \vdash ?\ \forall\, s, t : seq\ X \bullet items(s \frown t) = items\ s \uplus items\ t$$

L214 ==

$$[X] \vdash ?\ \forall\, B, C : \mathrm{bag}\ X \bullet dom(B \uplus C) = dom\ B \cup dom\ C$$

L215 ==

$$[X] \vdash ?\ \forall\, B : \mathrm{bag}\ X \bullet [\![\,]\!] \uplus B = B \uplus [\![\,]\!] = B$$

L216 ==

$$[X] \vdash ?\ \forall\, B, C : \mathrm{bag}\ X \bullet B \uplus C = C \uplus B$$

L217 ==

$$[X] \vdash ?\ \forall\, B, C, D : \mathrm{bag}\ X \bullet (B \uplus C) \uplus D = B \uplus (C \uplus D)$$

L218 ==

$$[X] \vdash ?\ \forall\, B : \mathrm{bag}\ X \bullet B \uminus [\![\,]\!] = B$$

L219 ==

$$[X] \vdash ?\ \forall\, B : \mathrm{bag}\ X \bullet [\![\,]\!] \uminus B = [\![\,]\!]$$

L220 ==

$$[X] \vdash ?\ \forall B, C : \mathrm{bag}\, X \bullet (B \uplus C) \uminus C = B$$

L221 ==

$$[X] \vdash ?\ \forall m, n : \mathbb{N};\ B : \mathrm{bag}\, X \bullet (n + m) \otimes B = n \otimes B \uplus m \otimes B$$

L222 ==

$$[X] \vdash ?\ \forall m, n : \mathbb{N};\ B : \mathrm{bag}\, X \mid n \geq m \bullet (n - m) \otimes B = n \otimes B \uminus m \otimes B$$

L223 ==

$$[X] \vdash ?\ \forall n : \mathbb{N};\ B, C : \mathrm{bag}\, X \bullet n \otimes (B \uplus C) = n \otimes B \uplus n \otimes C$$

L224 ==

$$[X] \vdash ?\ \forall n : \mathbb{N};\ B, C : \mathrm{bag}\, X \bullet n \otimes (B \uminus C) = n \otimes B \uminus n \otimes C$$

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